CALCULATING THE INTERPHASE RESISTANCE ON CONDENSATION OF PURE SATURATED VAPORS

N. V. Pavlyukevich, V. G. Leitsina, and G. E. Gorelik

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An approximate analytical solution is obtained for the problem of the condensation of pure saturated vapors, with consideration given to the interphase resistance for certain special cases. The results are compared with experimental data.

The classical Nusselt [1] solution for the film-condensation problem was subsequently refined by several authors [2-6].

The condensate temperature at the vapor–liquid interface was assumed to be equal to the vapor-saturation temperature in all of the above-cited references.

However, we know [13, 14] that on condensation of the vapors of any liquid the condensate surface is supercooled, i.e., the temperature difference between the saturated vapor and the condensate at the phase separation boundary is not zero. This temperature difference reflects the existence of interphase resistance, which becomes particularly pronounced in the case of the vapor condensation of liquid metals [7-9].

Reference [15] describes a simple graphoanalytical method of determining the temperature jump at the boundary of phase separation. The method of integral relationships was used in [9] to undertake a theoretical investigation of the condensation of liquid-metal vapors. A solution was achieved in [10] for the problem of water-vapor condensation at low pressures; however, as demonstrated in [11], an incorrect boundary condition was used to account for the interphase resistance.

The purpose of this paper is to dervie an analytical solution for the film-condensation problem as it pertains to saturated vapors and a vertical wall, without resort to the method of integral relationships.

Below we examine cases of vapor condensation, primarily at low pressures. The simplifications associated with neglect of the inertial forces and the supercooling resulting from convection are completely valid in this case. The problem of the condensation of a pure nonmoving saturated vapor on a vertical wall, with consideration of the interphase resistance, reduces to the following boundary-value problem (we neglect the effect of the shearing stresses at the boundary of phase separation):

$$\nu \frac{\partial^2 u}{\partial y^2} = -g, \tag{1}$$

$$\frac{\partial^2 T}{\partial y^2} = 0; \tag{2}$$

1) at the liquid–wall interface ($y = 0$)

$$u = 0, \quad T = T_\omega; \tag{3}$$

2) at the vapor–liquid interface ($y = \delta(x)$)

$$\frac{\partial u}{\partial y} = 0, \tag{4}$$

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TABLE 1. Theoretical Data on the Condensation of Water Vapor for $T_\infty - T_\omega = 10°$

p_∞, N/m²	T_∞, °K	$\delta_{Nu}\cdot 10^{-4}$ m	σ	$\delta\cdot 10^{-4}$ m	$T_0 - T_\omega$, °K	$\frac{A_1}{a}\cdot 10^{-4}$ m
101300	373,16	0,774	1	0,774	10	0,00059
			0,04	0,767	9,72	0,029
13300	324,74	0,891	1	0,891	10	0,0026
			0,04	0,859	8,95	0,129
1330	284,4	1,063	1	1,059	9,89	0,016
			0,04	—	—	0,77

$$\rho z L = \lambda \frac{\partial T}{\partial y}, \tag{5}$$

$$-\rho z = \frac{2\sigma}{2-\sigma}\left(\frac{p_0}{\sqrt{2\pi R T_0}} - \frac{p_\infty}{\sqrt{2\pi R T_\infty}}\right), \tag{6}$$

where

$$z = \frac{d}{dx}\int_0^\delta u\,dy\,, \tag{7}$$

Equations (1) and (2) with boundary conditions (3)-(5) have solutions of the form

$$u = \frac{g}{\nu}\left(\delta y - \frac{y^2}{2}\right), \tag{8}$$

$$T = T_\omega + \frac{\rho z L}{\lambda} y. \tag{9}$$

According to (7) and (8), for z we have

$$z = \frac{g}{\nu}\delta^2\frac{d\delta}{dx} = \frac{g}{3\nu}\frac{d}{dx}(\delta^3). \tag{10}$$

We have to use the remaining boundary condition (6) to determine $\delta(x)$. Here we have to distinguish two cases (I and II).

I. The temperature difference $T_0 - T_\omega$ across the condensate film is small (several degrees). This is the case in the condensation of liquid-metal vapors at low pressures. The pressure p_0 in (6) can then be expressed in terms of the known quantity p_ω (the saturation pressure, which corresponds to the wall temperature T_ω), linearizing the formula for the saturated-vapor pressure at the segment $T_0 - T_\omega$.

Having rewritten (6) to the form [12]

$$-\rho z = \frac{2\sigma}{2-\sigma}\frac{1}{\sqrt{2\pi R T_0}}\left[(p_0 - p_\infty) + \frac{p_\infty (T_\infty - T_0)}{2T_0}\right],$$

after linearization we have

$$T_0 = T_\omega + \frac{R T_\omega^2}{p_\omega L}\left[-\frac{2-\sigma}{\sigma}\rho z\sqrt{2\pi R T_\omega} - p_\omega + \frac{p_\infty}{2}\left(3 - \frac{T_\infty}{T_\omega}\right)\right]. \tag{11}$$

Having substituted (11) into (9), and using (10), we derive the equation for the determination of the film thickness $\delta(x)$

$$\frac{L\rho g}{4\nu\lambda}\frac{d}{dx}(\delta^4) + \frac{2-\sigma}{\sigma}\frac{R T_\omega^2 \rho g\sqrt{2\pi R T_\omega}}{6 p_\omega L \nu}\frac{d}{dx}(\delta^3) = \frac{R T_\omega^2}{p_\omega L}\left[\frac{p_\infty}{2}\left(3 - \frac{T_\infty}{T_\omega}\right) - p_\omega\right]. \tag{12}$$

Using the notation

TABLE 2. Comparison of Theoretical and Experimental Data on the Condensation of Mercury

p_∞, N/m²	T_ω, °K	T_∞, °K*	q, J/m²sec	$\delta \cdot 10^{-4}$ m	$\delta_{Nu} \cdot 10^{-4}$ m	σ	σ_{exp}	$T_0 - T_\omega$, °K	$(T_0 - T_\omega)_{exp}$, °K	$\frac{A}{a} \cdot 10^{-4}$ m	$\frac{\alpha_{Nu}}{\alpha}$
186,2	357,20	405,40	109322	0,424	1,419	0,522	0,516	0,49	0,39	176,2	28,08
1117,2	439,66	451,73	227250	0,523	0,996	0,535	0,530†)	1,20	0,89	7,565	5,29
2261	457,36	472,49	314010	0,580	1,054	0,377	0,365	1,79	1,28	7,411	7,64

*Given the same pressure p_∞, the temperature T_∞ is given somewhat lower here than in [8], since for the case described in that reference the vapor in the condensation chamber was slightly superheated.

†A value of 0.503 is given in [8], but calculations with the corresponding formula yield 0.530.

$$a = \frac{L\rho g}{4\nu\lambda},$$

$$A = \frac{2-\sigma}{\sigma} \frac{RT_\omega^2 \rho g \sqrt{2\pi RT_\omega}}{6p_\omega L\nu}, \tag{13}$$

$$B = \frac{RT_\omega^2}{p_\omega L}\left[\frac{p_\infty}{2}\left(3 - \frac{T_\infty}{T_\omega}\right) - p_\omega\right],$$

we write (12) in the form

$$\delta^4 + \frac{A}{a}\delta^3 = \frac{B}{a}x. \tag{14}$$

We can demonstrate that for temperature differences $T_\infty - T_\omega$ that are not too small (see Section III) in the case of some fixed value of x we have inequality

$$\frac{A}{a} \gg \delta \approx \left(\frac{B}{A}x\right)^{1/3}. \tag{15}$$

Considering (15), and using the small-parameter method, we find a solution for (14) in the form of the series

$$\delta = \left(\frac{B}{A}\right)^{1/3} x^{1/3}\left[1 - \frac{a}{3A}\left(\frac{B}{A}\right)^{1/3} x^{1/3} + \frac{a^2}{3A^2}\left(\frac{B}{A}\right)^{2/3} x^{2/3} - \ldots\right]. \tag{16}$$

It should be noted that the film thickness δ_0 in the zeroth approximation, according to (16), is proportional to $x^{1/3}$, whereas according to the Nusselt theory $\delta_{Nu} \sim x^{1/4}$.

Using (10) and (16), we can find the heat-flux density q at the wall, according to (5), and from (9) we can find the liquid temperature T_0 at the phase interface.

If (15) is not satisfied, i.e., both terms in the left-hand member of (14) are of the same order, we have to solve (14) by another method to determine $\delta(x)$.

II. Let us assume that the interphase resistance is small. As demonstrated by experiment and theoretical calculation, this is the case with water-vapor condensation, while when the saturation pressure is close to the atmospheric, this is also the case for the liquid-metal vapors. The pressure p_0 can thus be expressed in terms of the known pressure p_∞, linearizing the formula for the pressure of the saturated vapor at the segment $T_\infty - T_0$.

In this case we find a solution for the slightly complicated problem of saturated-vapor condensation on a vertical surface. We know [13] that in the cooling of the wall its surface temperature does not remain constant, since the cooling liquid is heated during the flow. We can account for the thermal resistance of a rather thin wall by treating the temperature T^* of its outside surface as a function of the coordinate x.

The system of equations for this conjugacy problem involves the equations of motion (1) and energy (2) in the liquid film, as well as the heat-conduction equations for the wall at which the condensation is taking place. Since the wall is rather thin ($h \ll x$), the temperature distribution within the wall and within the liquid film is sought in the form of a function that is linear with respect to y:

$$T_1 = b(x) + c(x)y,$$

$$T = d(x) + e(x)y.$$

Condition (6) is now written in the form

$$T_\infty - T_0 = \frac{2-\sigma}{2\sigma} \frac{\rho R T_\infty^2 \sqrt{2\pi R T_\infty}}{p_\infty L} z. \tag{17}$$

At the liquid–wall interface (y = 0)

$$T = T_1, \quad \lambda \frac{\partial T}{\partial y} = \lambda_1 \frac{\partial T_1}{\partial y}. \tag{18}$$

At the outside wall surface (y = −h)

$$T_1 = T^*(x). \tag{19}$$

Conditions (5), (18), and (19) are used to determine the functions b(x), c(x), d(x), and e(x).

Substituting (17) into (9), we find the equation for $\delta(x)$

$$\frac{d}{dx}(\delta^4) + \left(\frac{A_1}{a} + D\right)\frac{d}{dx}(\delta^3) = T_\infty - T^*(x).$$

Here a is calculated from (13), i.e.,

$$A_1 = \frac{2-\sigma}{\sigma} \frac{R T_\infty^2 \rho g \sqrt{2\pi R T_\infty}}{6 p_\infty L \nu},$$

$$D = \frac{4\lambda h}{3\lambda_1}.$$

Since a and A_1 are constants, we have

$$\delta^4 + \left(\frac{A_1}{a} + D\right)\delta^3 = \frac{1}{a}\int_0^x (T_\infty - T^*)\, dx. \tag{20}$$

Unlike (14), in (20) the coefficient for δ^3 is made up of two terms, one of which (A_1/a) reflects the existence of interphase resistance, while the other (D) reflects the thermal resistance of the wall.

Neglecting the wall resistance ($D = 0$, $T^* = T_\omega = \text{const}$), we find (20) in the form

$$\delta^4 + \frac{A_1}{a}\delta^3 = \frac{B_1}{a} x, \tag{21}$$

where $B_1 = T_\infty - T_\omega$.

We can demonstrate (Section III) that in a number of cases we have the inequality (for a fixed value of x)

$$\frac{A_1}{a} \ll \delta \approx \left(\frac{B_1}{a} x\right)^{1/4}. \tag{22}$$

Then, using the small-parameter method, we find the solution for (21) in the form of the series

$$\delta = \delta_0\left(1 - \frac{A_1}{4a\delta_0} + \frac{3A_1^2}{32a^2\delta_0^2} - \ldots\right), \tag{23}$$

where

$$\delta_0 = \left(\frac{B_1}{a} x\right)^{1/4}. \tag{24}$$

Consequently, the zeroth approximation for $\delta(x)$ in this case in the classical Nusselt solution.

If (22) is not satisfied, we must solve (21) in another way to determine $\delta(x)$.

It should be pointed out that even for the condensation of water vapor, to determine the film thickness we are not always able to limit ourselves to the zeroth approximation. We see from Table 1 that if the condensation factor $\sigma = 0.04$, even for p_∞ = 100 mm Hg we should take into consideration the second term in (23). Moreover, when $\sigma = 0.04$ and p_∞ = 10 mm Hg inequality (22) is not satisfied, i.e., both terms in the left-hand member of (21) are commensurate and the solution of (23) is not reliable.

It should be pointed out that the temperature jump at the interface, calculated from (16), is a weak function of the condensation factor σ, since the quantity z in zeroth approximation is inversely proportional to A. However, when the jump is calculated from (23), its value in zeroth approximation is proportional to $(2-\sigma)/\sigma$.

Let us return to (20), assuming $D \neq 0$ and $T^* = T^*(x)$. For mercury-vapor condensation, e.g., on a nickel surface the coefficient D is on the order of 10^{-3} m ($h \sim 10^{-2}$ m). Even for a vapor pressure p_∞ close to the atmospheric, when $A_1/a \sim \delta \sim 10^{-4}$ m (the interphase is substantially smaller than the wall resistance), instead of (23) we should therefore use (16), replacing A/a in the latter by $D + A/a$, and replacing B by $B_1(x) = \int_0^x (T_\infty - T^*)dx$.

This means that we can draw the conclusion to the effect that consideration of the thermal resistance of the wall somewhat expands the limits of applicability for a solution such as (16), as described in the previous section.

In the case of water vapor, when D is on the order of 10^{-5}-10^{-4} m ($h \sim 10^{-2}$ m), unlike the Nusselt expression (which for $D = 0$ yields good accuracy) we find the solution either in the form of a series as in (23) (replacing A_1/a by $A_1/a + D$), or by numerical solution of (21).

We can examine the boundary-value problem in which, in the place of the boundary condition $T|_{y=0} = T_\omega$ we use the condition

$$\lambda \frac{\partial T}{\partial y}\bigg|_{y=0} = q_\omega(x).$$

Bearing in mind (10), we derive the equation for $\delta(x)$

$$\delta^3 = \frac{3\nu}{L\rho g}\int_0^x q_\omega(x)\,dx.$$

III. In Table 2 the experimental data [8] on mercury-vapor condensation are compared with the theoretical data derived here. The theoretical values have been found on the basis of (16) for $\delta(x)$, and this formula is applicable to all the cases considered in [8], since the temperature difference $T_0 - T_\omega$ across the film does not exceed 3° and we have inequality (15). In calculating $\delta(x)$ we use the two first terms in (16). The heat-flux density q, calculated with consideration of the zeroth approximation exclusively, is compared with the experimental q_{exp} [8]. As a result, we find the zeroth approximation for the condensation factor σ_0:

$$q_{exp} = q\,\frac{\sigma_0}{2 - \sigma_0},$$

where q_1 is the value of the heat flow calculated for $\sigma = 1$.

Since σ is found in each term of (16), to refine σ we employ an iteration process (2-3 iterations). We see from Table 2 that the theoretical values are in good agreement with the experimental. The found values for the film thickness δ are within the limits the experimental values of [8].

Moreover, the table gives the results derived from the Nusselt theory (24), i.e., without consideration of the interphase resistance. We see easily that the values of the heat-transfer coefficient α_{Nu}, calculated on the basis of Nusselt theory, are considerably greater than the measured values (or those found here with consideration of the interphase resistance).

Table 1 shows the theoretical data on the condensation of water vapor for two values of the condensation factor: $\sigma = 1$ and $\sigma = 0.04$. The calculations were carried out on the basis of (23). These data are compared with the corresponding values, calculated on the basis of Nusselt theory.

NOTATION

p_∞ and p_0 are, respectively, the saturated-vapor pressures corresponding to T_∞ and T_0;
x and y are the coordinates, respectively, along the wall and along the normal to the wall;
u is the velocity component in the direction of the wall;
ρ is the liquid density;
ν and λ are, respectively, the coefficients of kinematic viscosity and thermal conductivity for the liquid;
L is the latent heat of vapor condensation;
σ is the condensation factor.

Subscripts

∞, 0, and 1 — pertain, respectively, to the parameters in the vapor, in the liquid at the vapor–liquid interface, and in the wall.

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